

CREATE SISO STATE SPACE MODEL OF MAIN SPINDLE FROM ANSYS MODEL

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Abstract: This paper shows identification of dynamic characteristics of working unit module main spindle, based on application of APDL ANSYS and MATLAB software. Single Input Single Output (SISO) state space MATLAB model will be developed from an APDL ANSYS main spindle model. The first step in defining the state space model is to define the eigenvector elements for all modes for only the input and output degrees of freedom. The next step is analysing the each mode contribution and sort them from the largest to smallest. One of the modal reduction technique is then applied, and modes with smallest contribution are simply truncated.

Key words: Main spindle, modal parameters, model size reduction, state space

Redukcija reda modela sklopa glavnog vretena metodom skraćivanja i analiza dinamičkog ponašanja u prostoru stanja. U radu je prikazana identifikacija dinamičkih karakteristika glavnog vretena modula radne jedinice pinolebazirana na primjeni APDL ANSYS i Matlab softvera. Primjenom metoda konačnih elementa utvrđeni su modalni parametri glavnog vretena, na osnovu kojih semodel konačnih elemenata transformiše u manje zahtjevan model, model prostora stanja, koji sa dovoljnom tačnosti opisuje dinamičko ponašanje modela. Na primjeru glavnog vretena prikazna je jedna od metoda redukcije reda, metoda skraćivanja, čijom primjenom se smanjuje računarska složenost modela i vrijeme simulacije.

Ključne riječi: Glavno vreteno modula pinole, modalni parametri, redukcija reda modela, prostor stanja

1. INTRODUCTION

One of the biggest obstacle in work with finite element models is system size. During a sequence of FEM analysis of complex mechatronic system like machine tool is, nodal displacement are computed for mesh which can have up to 10^5 , even 10^6 degree of freedom (DOF). Considering that design process is very interactive, so FEM analysis needs to be repeated frequently, it is obvious that perform finite element analysis repeatedly can be very time consuming. Although modern computers are becoming more and more powerful, dealing with such huge models is still a problem. Model order reduction (MOR) techniques reduces the size of the FEM model while input/output relationship remains preserved.

Using modal analysis eigenvalues (resonant frequencies) and eigenvectors (mode shape) are determined. Since the number of eigenvalues and eigenvectors is equal to number of DOF, it is obvious that this is too large to be inserted into state space in MATLAB model. Therefore, the goal to be achieved is not only obtaining a reliable dynamic model, since such a model exist and that is a FEM model. The objectives to be achieved are:

- Convert a large finite element model (by “large model” it is assume model with thousands of hundreds DOF) to a smaller MATLAB model which still provide correct response for the forcing input, i.e. still maintaining the input / output relationship, Fig. 1.,
- Obtained MATLAB model can be inserted into complex control system models and be used do define system dynamic.

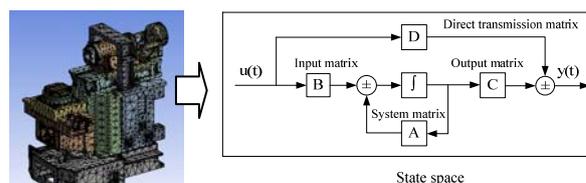


Fig. 1. Model size reduction

The mathematical model obtained by finite element method is linear time-invariant model. The entire structure is described by the mass, damping and stiffness matrices and can be represented by linear ordinary differential equations with constant coefficients. This model can be transformed into state space representation where a system an n^{th} order differential equation is replaced with a single first order matrix differential equation.

When carrying out certain analysis, such as for example modal analysis, considered system has a highly complex mathematical model which solving can take a long time. In order to reduce the time required for the execution of the simulation, still maintaining the input / output relationship, different model reduction technique are applied. Maglie [3] used the FEM model to determine the relative movement of the tool tip to the workpiece under the influence of the cutting force. Maglie [3] presented a platform for the exchange of data between ANSYS WORKBNCH and simulation model developed in MATLAB / Simulink in which further static and dynamic analysis are performed. The platform integrates modules that contain characteristics of joints, while set of macros is exporting models from ANSYS WORKBNCH into MATLAB / Simulink, including the properties of the joints, stiffness and

damping. Maglie [3] used MOR for ANSYS, commercial model order reduction software tool for reading ANSYS files and performing Krylov-based model reduction.

Vesely [5] analysed the impact of natural frequencies of the machine tool mechanical structure on the dynamic behaviour of the feed drive. Machine tool structure is being observed in two ways: as lumped mass in one, and as a finite element model in the second case. Machine tool is modelled as a system of elastic bodies, and model order reduction of FEM model is performed by applying Craig – Bamton reduction techniques. This technique allows the selection of a certain number of natural frequencies the reduced model will keep comparing to the original model, and the selection of so-called "interface" nodes that are used for coupling with other bodies. Also, Vesely [5] analysed two methods for determining the influence of the machine tool structure on the dynamic behaviour, the first way - modal decomposition technique in which the transformation is performed on mass, damping and stiffness matrix obtained from the FEM system, and otherwise - by solving the equation of motion in modal coordinates.

2. MAIN SPINDLE MODAL ANALYSIS

In this paper, on the main spindle example, Fig. 2., is shown how the model can be transformed from finite elements into state space representation, how to do modal reduction sorting modes by dc gain and peak gain, perform sorted modes modal truncation and finally find solution in physical coordinates through the next steps, [4]:

1. Perform modal analysis, and determination of natural frequencies and mode shapes in ANSYS (eigenvalues / eigenvectors),
2. Transformation of the FEM model so that the modal matrix include only those degrees of freedom where the force is applied and/or outputs desired,
3. Analysing the modal contribution of each mode and sort them in descending order, as well as reducing the number of modes by modal truncation including only modes which have significant contribution to desired response,
4. Create modal state space form equations of motion,
5. Find solution in physical coordinates in frequency or time domain.

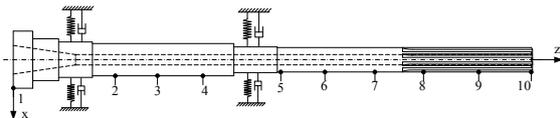


Fig. 2. Equivalent dynamic model of spindle with measurement points

Main spindle, shown in Fig 2., is supported by the two sets of angular contact ceramic ball bearings in front, SKF S7011 CD/HCP4A and two sets of angular contact ceramic ball bearings in rear SKF 7008 CD/HCP4A, installed back to back, Fig 2. Values of

bearing stiffness depending on the preload are provided in the table 1 [6], [2].

	Preload					
	Small		Middle		Big	
	Fron t brg set	Fron t brg set	Fron t brg set.	Fron t brg set	Fron t brg set	Front Brg set
Radial stiffness N/ μ m	400	280	560	387	690	475
Axial stiffness N/ μ m	69	48	115	82	170	120

Table 1. Stiffness of bearing [6], [2]

Considering that quill unit moves horizontally (feed movement), the main spindle assembly has a specific construction, with free end of the main spindle relatively large length, [6], [7]. Therefore, it is especially interesting to consider the dynamic behaviour of the main spindle free end. Fig. 3. shows main spindle modelled in APDL ANSYS with two elastic support on the bearing places. The material is linear isotropic structure steel. Young's modulus is $E=2 \cdot 10^{11}$ Pa, Poisson's ratio is $\nu=0.3$, material density $\rho = 7850$ N/m³. SOLID186 a higher order 3-D 20-node solid element is used to simulate main spindle. Total numbers of nodes and elements are 85032 and 17167 respectively. When assigned material to model in ANSYS, the spindle mass is 9.2 kg. Spring damper element COMBIN 14 is applied to simulate the elastic support of the two set of bearings. Eighteen elements was set along the circumferential direction of the spindle on each set of bearing, simulating rolling elements. Since the inertial force and the thermal expansion of bearing elements affect the balls, an uneven distribution of contact forces and an uneven contact angle change occurs. The consequence of the aforementioned is the uneven distribution in bearing stiffness, depending on the position of the ball [7].

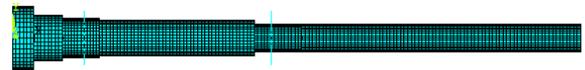


Fig. 3. Main spindle finite element model

The first ten eigenvalues (Tab. 2) and eigenvectors for bending motion of main spindle were extracted using Block-Lanczos method. Each of 10 column of modal matrix represent the eigenvector for that mode, normalized with respect to mass. Each row of the modal matrix show relative motion of points to which forces are applied and/or for which displacement are desired (measurement points from Fig. 2).

f_1 (Hz)	f_2 (Hz)	f_3 (Hz)	f_4 (Hz)	f_5 (Hz)
154,37	926,92	1641,06	2351,83	2538,17
f_6 (Hz)	f_7 (Hz)	f_8 (Hz)	f_9 (Hz)	f_{10} (Hz)
4533,14	5687,47	6972,75	8228,99	9637,48

Table 2. Eigenvalues for bending motion of main spindle, Hz

3. MAIN SPINDLE STATE SPACE REPRESENTATION

In the state space formulation, n second order differential equations are converted to 2n first order differential equations. The most general state-space representation of a linear system is

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx + Du\end{aligned}\quad (1)$$

where first equation is state equation, and second output equation, and x is the state vector, u is the input (or control) vector, y is the output vector, A is the state (or system) matrix, B is the input matrix, C is the output matrix, D is the "feedthrough (or feedforward) matrix" (in cases where the system model does not have a direct feedthrough, D is the zero matrix.

Rewriting the equation 3.1 in matrix form as:

$$\begin{aligned}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\omega_2 & -2\zeta_2\omega_2^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ F_{p1} \\ 0 \\ F_{p2} \end{bmatrix} \\ \dot{x} &= A x + Bu\end{aligned}\quad (2)$$

It may be noted that along the diagonal two sets of uncoupled first order equations appear in block of 2x2 coefficients, where the first block 2x2 represent response of the first mode and second block represent response second mode.

$$\begin{aligned}\text{Response of the first mode:} & \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \\ \text{Response of the second mode:} & \begin{bmatrix} 0 & 1 \\ -\omega_2 & -2\zeta_2\omega_2^2 \end{bmatrix}\end{aligned}\quad (3)$$

Damping ratio is known, either to be adopted as a single value for all modes, or column vector whose members are individual damping ratio for each mode. The natural frequencies (eigenvalues) are also known. This means that the process of determining the submatrix A can easily be expressed algorithmically, and thus the programmed. Uncoupled equation of motion of two first modes are

$$\begin{aligned}\text{First mode} & \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ F_{p1} \end{bmatrix} u \\ \text{Second mode} & \begin{bmatrix} \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_2 & -2\zeta_2\omega_2^2 \end{bmatrix} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ F_{p2} \end{bmatrix} u\end{aligned}\quad (4)$$

The transformation of the dynamic behaviour of finite elements in the state space involves performing two types of reduction: reducing the number of DOF of the FEM model and the modal reduction.

Reducing the number of DOF of the FEM model means that new model analysed in MATLAB include only the degree of freedom to which forces are applied and/or for which displacement are desired.

If the new model still comprises a number of degrees of freedom bigger than needed, then modal reduction based on the set of criteria is carried out. Analysis of the modal contributions of each individual modes and sort them according to the relative importance is the criterion for performing modal reduction.

The procedure is the following: modes are first ranked on the basis of their relative importance, and then performs the elimination of modes with a lower value modal contributions, i.e. **modal truncation**. Parameters on which determines the sorting mode is the **damping ratio** ζ . The criteria to be applied for sorting modes depends on whether the value of the damping ratio ζ considered unique or different for each mode.

In the first case the criterion for ranking modes is "**dc gain**", (3.5), while in the second case, the criterion used for ranking modes is "**pick gain**" (3.6) [1].

$$\frac{z_j}{F_k} = \sum_{i=1}^m \frac{z_{nji} z_{nki}}{\omega_i^2}\quad (5)$$

$$\frac{z_{ji}}{F_{ki}} = \frac{-j}{2\zeta_i} \left(\frac{z_{nji} z_{nki}}{\omega_i^2} \right)\quad (6)$$

Comparing (3.2) and (3.3) can be seen that peak gain is -90° phase shift at resonance, but to have greater amplitude for then dc gain. If relative damping has unique value for all modes, peak gain has larger amplitude but relative amplitude are the same. That means with unique relative damping ζ both methods give the same order of modes regardless which of the ranking criterion is applied. If relative damping has different values for all modes, then peak gain ranking criterion needs to be applied.

Usually the value of damping ratio is between 0,005 (0.5% of the critical damping) to 0.02 (2% of the critical damping). If the experimental results are available, i.e. real and imaginary part of the transfer function, damping ratio ζ can be determined for each mode, and in that case the obtained values of damping ratio are placed in a separate text file.

Fig. 4. shows dc gain value for all modes versus mode number for direct frequency response function (FRF) measurement location (X1/F1), and Fig. 5. for cross FRF (X10/F1). If the response is measured at the same location where force is applied then such FRF is called a direct, but if is measured at different location, this is cross FRF.

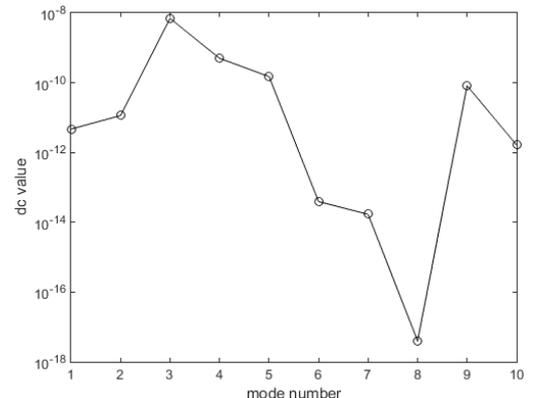


Fig. 4. dc value of each mode contribution versus mode number, direct FRF (X1/F1)

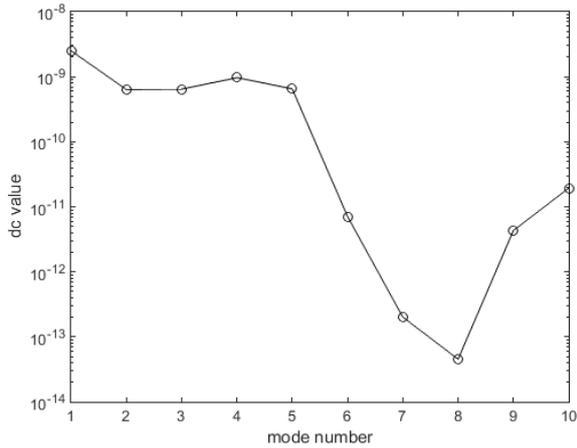


Fig. 5. dc value of each mode contribution versus mode number, cross FRF (X10/F1)

In this case constant damping ratio $\zeta = 0.001$ was considered as unique for all modes, so dc gain was used as a criterion for mode ranking. It can be seen that the mode 1 and 2 (frequencies 154.37 Hz and 926.92 Hz) have smaller modal contributions of modes 3,4 and 5. Accordingly, Fig. 6. and Fig. 7. show that the first two frequencies have such low gain that their resonant peaks are barely visible on the overall response.

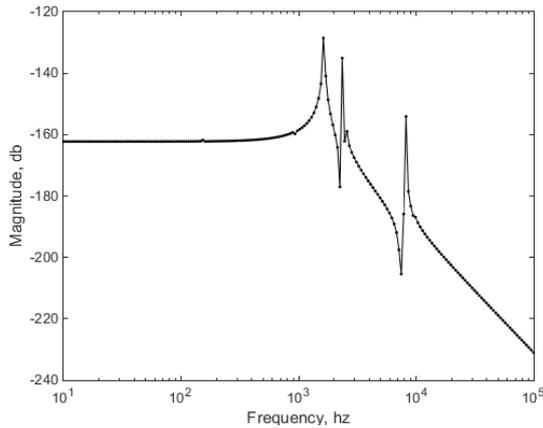


Fig. 6. Direct FRF, all 10 modes included

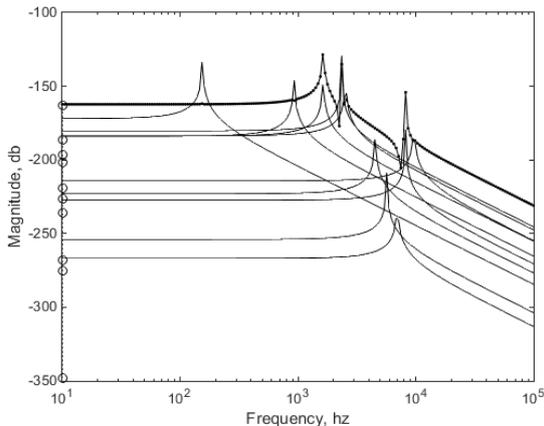


Fig. 7. Direct FRF, overlay of each individual mode contributions

Fig.7 show the overall frequency response with overlaid SDOF (Single Degree of Freedom) response of all the individual modes. Fig. 8. shows overall mode contributions for four selected modes. It can be seen

that modes 3,4,5 and 9 have the most significant modal contribution.

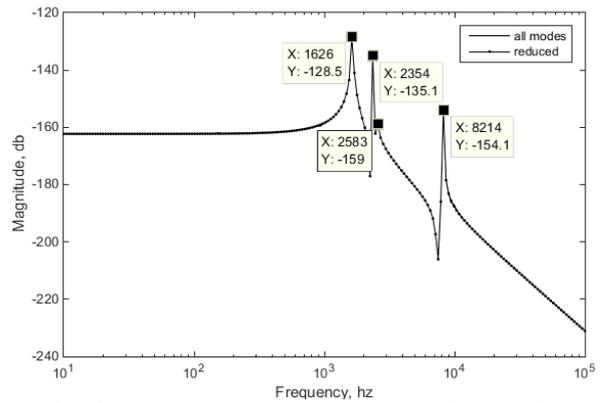


Fig. 8. Direct FRF – reduced sorted modal truncation frequency response –four modes included

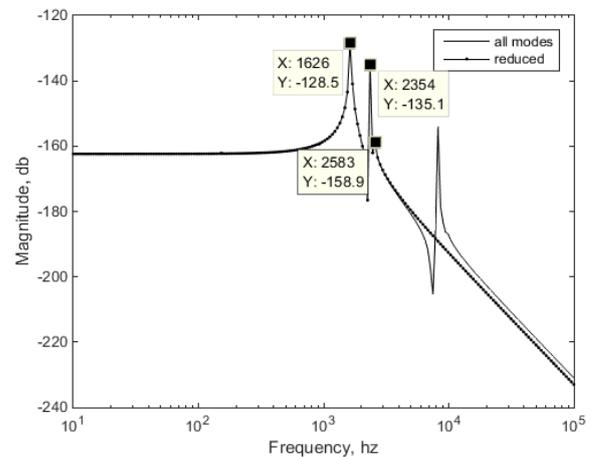


Fig. 9. Direct FRF – reduced sorted modal truncation frequency response –three modes included

If transfer function is plotted for three selected modes, it can be seen that the most significant modes are modes 3,4 and 5, Fig. 9. However, in the case of cross FRF, modal contribution of the first four modes is significantly higher than in other modes, Fig.10. Fig.11. shows overall mode contribution for all ten modes and four sorted modes for cross FRF (X10/F1). It can be noted that it is necessary to create at least two different state space models, the first includes all modes extracted from ANSYS, and second only desired number of modes, i.e. reduced number of modes.

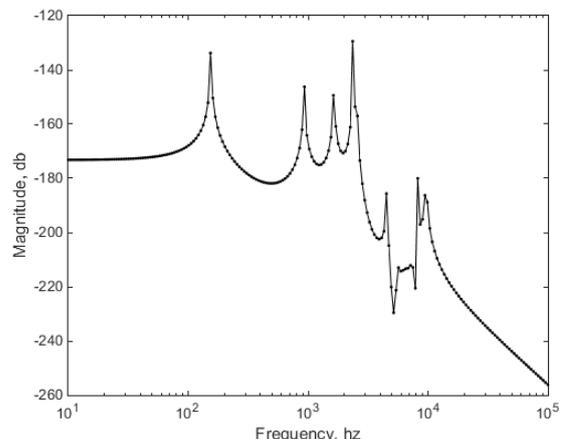


Fig. 10. Cross FRF (X10/F1)– all ten modes included

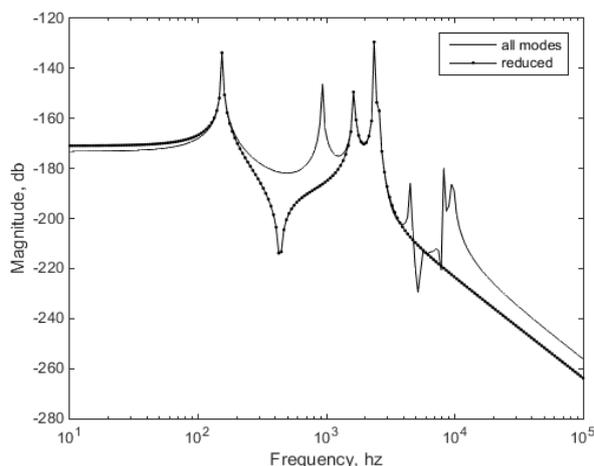


Fig. 11. Cross FRF (X10/F1), reduced sorted modal truncation, 4 modes included

From Fig. 11. can be seen that there is an error in the low frequency portion of the frequencies response. This is because the influence of truncated modes on dc gain did not take into account. This influence can be replaced using other techniques of modal reduction, which will be the subject of further research.

4. FINAL REMARKS

This paper shows how to take the results of finite element main spindle model created in ANSYS and reduce the model size extracting lower order state space model in MATLAB (model reduction). Then number of modes can be reduced (modal reduction) by ranking relative importance of each mode to overall response. It was shown modal truncation, as one of techniques for modal reduction. A reduced solution provides very reliable dynamic of the model with a significant reduction in number of states.

Reduced mode can be further inserted into a more complex control system model and used to find system dynamics.

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