EMPIRICAL MODELLING OF DYNAMIC FORCES AND PARAMETER OPTIMIZATION USING TEACHING-LEARNING-BASED OPTIMIZATION ALGORITHM AND RSM IN HIGH SPEED BALL-END MILLING

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Abstract: In the present paper, dynamic cutting force components have been modelled using response surface methodology based on design of experiments. Ball end milling tests have been performed according the experimental plan using central composite design. Analysis of variance (ANOVA) has been performed to test for adequacy on the experimental data and a full quadratic mathematical models have been established. ANOVA for the individual terms revealed that axial depth of cut is the most dominant cutting parameter for tangential and axial dynamic cutting forces, accounting for 49.27% and 45.10% contributions, respectively. Radial depth of cut is the most dominant parameter for radial force and contributes 64.21% for it. Composite desirability function and teaching learning based optimization (TLBO) algorithm have been used for determining optimal cutting process parameters like of feed per tooth, cutting speed, axial and radial depth of cut. Optimum values of cutting parameters have been obtained using composite desirability function (CD) and TLBO. The best optimum value of cutting process parameters has been obtained through TLBO and are feed per tooth \((f_z) = 0.06\) mm, axial depth of cut \((a_p) = 0.79\) mm, cutting speed \((V) = 169\) m/min, and radial depth of cut \((a_e) = 0.42\) mm. The optimum value of dynamic cutting force components have been validated by conformation experiments and are in good agreement with the predicted result.

Key words: Ball end milling, dynamic cutting force, optimization, response surface methodology, TLBO

1. INTRODUCTION

High speed ball end milling is a machining process used extensively in automobile, aerospace and, die and mould making industries. This is a very efficient machining process for producing aerodynamic sculptured components like turbine blade and propellers with high dimensional accuracy and surface finish. Machinability, dimensional accuracy and tool deflection are highly influenced by cutting forces in high speed ball end milling. In addition, cutting forces play a major role in determining tool life, surface finish and machine tool vibration. Thus, a precise knowledge of cutting force is very much essential for an effective ball end milling process. In high speed machining process, high cutting speed results in highly localized temperature resulting in thermal softening of the workpiece material [1]. In addition, a high speed leads to decrease in chip thickness. This allows always a greater shear deformation and results in corresponding decrease in cutting force [2]. In ball end milling process, high cutting performance may be achieved by selecting appropriate cutting parameters. In industries, desired cutting parameters are usually determined based on the experience of the machining operators or by the use of handbook. Experience based cutting parameters do not ensure that the cutting performance is near optimal. Several mathematical models [3-9] have been proposed to select the appropriated cutting parameters to establish the relationship among cutting performances and cutting parameters based on statistical regression techniques like response surface
methodology (RSM), Taguchi method, and factorial methods or neural network. These mathematical model (models) has been further used as an objective function with constraints to solve for the optimal cutting parameters using optimization methods.

M. Alauddin et al. [3] attempted to model the cutting force for studying the effect of feed per tooth and axial depth of cut on average tangential cutting force in end milling of Inconel 718. It was found that both the feed and axial depth of cut affected the average cutting force. It increased with an increase in axial depth of cut or feed or both of them simultaneously. Ding et al. [4] also reported that cutting force components were mostly affected by axial depth of cut and feed in hard milling of AISI H13 steel. They had used maximum absolute value of cutting forces in one cutting period (i.e. one full rotation of the tool) for analysis using Taguchi method and concluded that a linear model could be fitted for all the three components of forces, namely axial force, feed force and normal force. Ozel et al. [5] employed four factor two level fractional factorial experiments to study the effect of cutting edge geometry, workpiece hardness, feed and cutting speed on surface roughness and cutting forces in hard turning of hardened AISI H13 steel. They found that all the above parameters were statistically significant. They reported that decrease in edge radius and workpiece hardness resulted in lesser tangential and radial forces. Dikshit et al. [6] carried out an experimental investigation to study the effect of feed per tooth, cutting speed, radial depth of cut and axial depth of cut in ball end milling of Al2014-T6 and reported that tangential, radial and axial cutting force components were mostly affected by axial depth of cut only.

C. K. Toh [7] studied the effect of cutter path strategies on tool life using the static and dynamic cutting forces. He concluded that, the cutting forces increased and tool life decreased with increase in axial depth of cut regardless of the milling path orientations. He also found that dynamic cutting forces were more sensitive to tool wear. Tangjitsitcharoen et al. [8] employed the dynamic cutting force ratio to predict in-process surface roughness using multiple regression analysis and found that increase in dynamic cutting force ratio increased surface roughness. Bhogal et al. [9] investigated the effect of process parameters on surface roughness and tool vibration in end milling using RSM. Dynamic cutting forces are very important in machining process because these are directly related to machine tool vibration, tool deflection and tool chattering, poor surface finish and premature tool failure. Dimla [10] established inter-relationship between vibration signals and the cutting forces to find out the dynamic nature of the cutting process. He reported that dynamic cutting forces have some impacts on the dynamic behavior of a cutting process. The developed model was further utilized for the development of tool condition monitoring systems (TCMSs).

In the present paper, mathematical models have been developed using RSM for dynamic cutting force components in high speed ball end milling to investigate the effect of process parameters viz. feed per tooth \(f_t\), cutting speed \(V\), radial depth of cut \(a_r\) and axial depth of cut \(a_d\). Machining experiments have been planned using design of experiments (DOE) based on response surface methodology. The analysis of variance (ANOVA) has been performed to investigate the effect of cutting process parameters on the dynamic forces in tangential \(F_x\), radial \(F_y\) and axial \(F_z\) directions. These mathematical models have been further used for optimization of dynamic cutting force components using meta heuristic optimization method called teaching learning based optimization (TLBO) and composite desirability function (CD). The optimal cutting parameters have been.

2. MATERIALS AND METHODS

2.1 Static and dynamic cutting forces

In ball end milling, time duration of tooth contact with the workpiece is a fraction of spindle period (time taken to complete one revolution of the tool or spindle) and the chip thickness varies over a revolution of the cutting tool. Force signals recorded by dynamometer are dynamic in nature due to the varying chip thickness and the nature of tool engagement with the workpiece. Post process of the collected force signals of each repeated test revealed that a section of force signal was constant and did not change with time as the cutting process progressed. This constant force signal was reported as static force signal [10] and was given by mean value of the sampled data as follows:

\[
F_S = \frac{1}{N} \sum_{i=1}^{N} F_i(t) \quad (1)
\]

where \(F_S\), \(N\), \(F_i\) are static force components, number of sample points and force signal data at \(i^{th}\) position, respectively. The cyclic variation of the force components beyond the threshold may result in dimensional inaccuracy and machine tool chatter. It is difficult to eliminate this cyclic variation as well as to predict the cutting conditions under which this cyclic variation occurs. Therefore, to get an indication of the system fluctuations, the dynamic forces are required to be studied carefully. Dynamic force signal can be found by subtracting static force from the maximum recorded force. If \(F_D\) is dynamic force component and \(F_{max}\) is the maximum force, then the dynamic force component may be mathematically expressed as:

\[
F_{D,XYZ} = F_{max,XYZ} - F_{S,XYZ} \quad (2)
\]

Dynamic cutting forces fluctuate with excursions to zero and then to higher magnitudes during cutting. The consequence of these excursions have been the onset of tool holder vibration whereby chattering often results at high magnitudes [11, 12]. For illustration, static and dynamic forces are shown in the Fig. 1. In the present paper, the dynamic cutting forces in \(F_x\), \(F_y\) and \(F_z\) directions have been determined using Eqs. (1 and 2) using 20000 sample points. Analysis of variance (ANOVA) and regression analysis have been performed using response surface methodology [13].
2.2 Experimental plan

Response surface methodology (RSM) is a sequential procedure and is a collection of statistical techniques used for modelling and analysis of processes in which a response of interest is correlated with several input variables. The objective of RSM is to determine the optimum operating condition in terms of a region of factor space in which output requirement(s) are satisfied. In this methodology, a response variable $Y$ is postulated to be a random variable and $k$’s number of input variables ($x_1$, $x_2$, $x_3$, ..., $x_k$) are presumed to be continuous. On performing experiments, it is possible to represent the functional relationship between independent input factors and response variable in quantitative form by regression analysis. Thus, mathematically, it may be written as:

$$Y = f \left( x_1, x_2, x_3, ..., x_k \right) + \varepsilon$$  \hspace{1cm} (3)

If the system has curvature, then a polynomial of higher degree must be used. A second order polynomial in independent variables is given by:

$$Y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_i x_i^2 + \sum_{i=1}^{k} \sum_{j=1}^{k} \beta_{ij} x_i x_j + \varepsilon$$  \hspace{1cm} (4)

where, $\beta_0$ is constant, $\beta_i$, $\beta_{ij}$, $\beta_{ij}$ are the constants of linear, square and interaction terms respectively. $\varepsilon$ is the experimental error. A central composite design (CCD) is the most popular design of experiment used for fitting the second order models. In the present study, CCD is used to carry out the experiments. The response variables considered are dynamic cutting force components in $F_X$, $F_Y$, and $F_Z$ directions. Four cutting parameters, i.e., $f_c$, $a_p$, $a_e$ and $V$ have been considered. The coded values along with actual values of these input variables are shown in the Table 1. The coded values of intermediate levels are obtained by:

$$X_i = \frac{2X - (X_{\text{max}} + X_{\text{min}})}{X_{\text{max}} - X_{\text{min}}}$$  \hspace{1cm} (5)

where $X_i$ is the required coded value of the variable $X$. $X$ is any value of variable lies between $X_{\text{min}}$ to $X_{\text{max}}$ i.e. maximum and minimum values of cutting parameters.

<table>
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<th>No. Factors</th>
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<tr>
<td>$a_p (X_2)$</td>
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<tr>
<td>$a_e (X_3)$</td>
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</tr>
<tr>
<td>$V (X_4)$</td>
<td>1 75 100 125 150 175</td>
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</table>

Table 1. Experimental parameters and their levels

3. OPTIMIZATION METHODS

3.1 Teaching learning based optimization (TLBO)

TLBO is a meta-heuristic optimization algorithm based on teaching–learning process developed by Rao et al. [14]. This is a very efficient and accurate nature inspired optimization technique. Rao et al. [14] had applied TLBO on different mechanical design benchmark problems like design of pressure vessel, tension/compression spring, welded beam and gear train etc. The effectiveness of TLBO was compared with the research available on the above mentioned problems using different optimization techniques. They had also employed TLBO on constrained mechanical design problem available in literature and reported that the performance of TLBO is better than the other nature inspired optimization methods for the constrained mechanical design problems.

Rao et al. [15] employed TLBO on non-linear optimization problems. They had compared the TLBO and other optimization techniques like genetic algorithm (GA), particle swarm optimization (PSO), artificial bee colony (ABC) and harmony search (HS). The effectiveness of TLBO was checked on different performance criteria viz. mean solution, success rate, average function evaluations and convergence rate etc. It was reported that TLBO has better performance over other optimization techniques stated earlier. Rao and Kalyankar [16] successfully applied TLBO on the examples previously attempted by various researchers for parameter optimization of different modern machining processes like, ultrasonic machining, abrasive jet machining, wire electrical discharge machining, etc. They had also compared the results obtained by TLBO to other nature-inspired optimization techniques and reported that considerable improvement was observed in results and convergence using TLBO. Rao et al. [17] had also applied TLBO for parameter optimization of a few selected casting processes like, squeeze casting, continuous casting and die casting. They had observed that results obtained from TLBO was satisfactory and requires less computational efforts. Baghlanl and Makiabadi [18] employed TLBO for shape and size optimization of truss structures with dynamic frequency constraints. Considering various benchmark problems they concluded that very satisfactory results were obtained using TLBO in comparison to those of PSO, HS and firefly algorithm (FA).

TLBO is a population based method and global optimum is obtained by using the population of solutions. TLBO is based on the effect of influence of a teacher on the output of learners in a class. The algorithm mimics teaching-learning ability of teacher and learners in a class room. In TLBO the population is
considered as a group of learners (students). Teacher and learners are the two vital components of the algorithm describing two basic modes of the learning, through teacher (known as teacher phase) and interacting with the other learners (known as learner phase). Thus, the algorithm is based on the teaching-learning ability of teacher and learners in a classroom and is distinguished in two parts namely (1) teacher phase and (2) learner phase.

### 3.1.1 Teacher phase

In a class room the teacher tries to increase the level of the knowledge of the students up to their level. But in actual practice it is not possible (extremely difficult) to increase the knowledge level of the students to a desired level. Therefore, the teacher tries to move the mean result of the class up to some extent (close to desired level) in the specific subject taught by him or her depending on the capabilities of the class [14]. Let us suppose that number of subjects also called design parameters is represented by \( m \), and \( n \) is the number of learners (students) i.e. population size. A teacher is treated as a highly knowledgeable who teaches learners for better results. Therefore, a teacher is considered as the best learner and hence tries to improve its mean results. At any particular iteration \( i \), \( M_{j,i} \) is the mean result of the learners in a particular subject \( j \). The best overall result \( X_{total-teacher} \) obtained in the entire population of learners considering all the subjects together can be considered as the result of best learner \( k_{teacher} \). The solution is updated according to the difference between the existing mean result of each subject and the corresponding result of the teacher for each subject and can be given by:

\[
\text{Difference } \_ \text{Mean}_{j,k,i} = r_i \left( X_{j,teacher} - T_F M_{j,i} \right) \quad (6)
\]

where, \( X_{j,teacher} \) is the result of the best learner (i.e., teacher) in subject \( j \), \( T_F \) is the teaching factor and \( r_i \) is the random number varies over the range of \([0, 1]\). The value of \( T_F \) is decided randomly with equal probability as [15]:

\[
T_F = \text{round} \left[ 1 + \text{rand}\left(0,1\right) \{2-1\} \right] \quad (7)
\]

\( T_F \) is not a parameter but a heuristic step in the TLBO algorithm. The best optimal solution using TLBO can be obtained if the value of \( T_F \) is either 1 or 2 and decided randomly by the algorithm using equation (7), [16]. The existing solution is updated in the teacher phase using the difference mean according to the following expression shown in equation (8):

\[
X'_{j,k,i} = X_{j,k,i} + \text{Difference } \_ \text{Mean}_{j,k,i} \quad (8)
\]

Where, \( X'_{j,k,i} \) is the updated value of \( X_{j,k,i} \). If \( X'_{j,k,i} \) gives a better function value, then it has to be accepted. All the accepted function values at the end of the teacher phase are maintained and these values become the input to the learner phase.

### 3.1.2 Learner phase

The knowledge of the learners can be increased through input from the teacher and through interaction among themselves. A learner can learn new things and enhance his or her knowledge with interacting with another learner which has more knowledge [14]. For a population size of \( n \), let us consider two learners \( P \) and \( Q \). The learning procedure of \( P \) and \( Q \) can be obtained by the updated solution of \( P \) and \( Q \) i.e. \( X_{total-P} \) and \( X_{total-Q} \) at the end of the teacher phase under the condition that \( X_{total-P,i} \neq X_{total-Q,i} \):

\[
X'_{j,p,i} = X_{j,p,i} + \eta \left( X_{j,p,i} - X'_{j,Q,i} \right) \quad \text{if } X_{total-P,i} < X_{total-Q,i} \quad (9)
\]

\[
X'_{j,p,i} = X_{j,p,i} + \eta \left( X_{j,p,i} - X'_{j,Q,i} \right) \quad \text{if } X_{total-P,i} > X_{total-Q,i} \quad (10)
\]

\( X'_{j,p,i} \) is accepted if it gives a better functional value. All the accepted functional values obtained at the end of the learner phase restored and these values become the input to the teacher phase of the next iteration. The random values \( (r_i) \) used in equations (6), (9), and (10) may be different. Equations (9) and (10) are used for a maximization problem. The reverse is also true for the minimization problem [16]. Pseudo code for TLBO for step by step demonstration is given below.

Set \( k=1 \)

Objective function \( f(X_1, X_2, X_3, X_4) \)

Generate initial population i.e. no of students

Calculate the objective function for all population while (the termination conditions are not met)

\{Teacher Phase\}

Calculate the mean of each design variable \( M_{j,i} \)

Identify the best solution (teacher)

for \( 1:n \)

Calculate teaching factor \( T_F = \text{round} \left[ 1 + \text{rand}\left(0,1\right) \{2-1\} \right] \)

Modify solution based on best solution (teacher)

\[
X'_{j,k,i} = X_{j,k,i} + \eta \left( X_{j,teacher} - T_F M_{j,i} \right)
\]

Calculate objective function for new mapped learner \( f(X'_{j,k,i}) \)

if \( X'_{j,k,i} \) is better than \( X_{j,k,i} \)

\( X_{j,k,i} = X'_{j,k,i} \)

end if {End of Teacher Phase}

\{Learner Phase\}

Randomly select another learner \( X'_{j,i} \) such that \( X'_{total-P,i} \neq X'_{total-Q,i} \)

if \( X'_{total-P,i} < X'_{total-Q,i} \)

\[
X'_{j,p,i} = X'_{j,p,i} + \eta \left( X'_{j,p,i} - X'_{j,Q,i} \right)
\]

else

\[
X'_{j,p,i} = X'_{j,p,i} + \eta \left( X'_{j,Q,i} - X'_{j,p,i} \right)
\]

end if

if \( X'_{total-P} \) is better than \( X'_{total-Q} \)

\( X'_{total-P} = X'_{total-Q} \)

end if {end of learner Phase}

Set \( k=k+1 \)

end while
3.2. Response surface optimization.

Response surface optimization is very useful to find out the cutting parameters (f, Aa, Aa, V) at which the response (FX, FY, and FZ) reach to the optimal value in high speed ball end milling process. Optimization using RSM may be divided into three groups, viz., (1) approach based on overlapping of contours, (2) composite desirability function and (3) dual response system methodology [19]. Composite desirability is one of the most widely used methods in industry which is based on weighted geometric mean of the individual desirability’s for the responses on a range from zero to one. The common approach is to find out the individual desirability index of the responses by transforming the corresponding response yi into the individual desirability function di(yi) varying over the range 0 ≤ di(yi) ≤ 1 using the suitable formulae as proposed by Derringer and Suich [20]. Ideal case is represented by 1 and 0 indicates the worst case, i.e., one or more responses are outside the desired limits. In this approach, the inputs are target value (Ty) upper value (ymax) and lower (ymin) value of the responses. There are three forms of desirability functions, viz., (1) the lower-is-the-better, (2) the higher-is-the-better and (3) the target-is-the-best, depending on whether a particular response problem yi is to be minimized, maximized or is assigned a target value. In the present study lower-is-the-better form is applied for minimizing FX, FY, and FZ which may be given as:

\[
di(yi) = \begin{cases} 
1 & y_i \leq y_{\text{min}} \\
\frac{y_i - y_{\text{min}}}{T_y - y_{\text{min}}} & y_{\text{min}} < y_i < y_{\text{max}} \text{, } r \geq 0 \\
0 & y_i \geq y_{\text{max}} 
\end{cases} \quad (11)
\]

Where, ymin, ymax, and Ty represents lower value, upper value and target value respectively of the response yi. ‘r’ indicates the weight and its value is larger if the response is close to the target value, otherwise, it is set to the lower value. It is the most important parameter that determines the shape of the desirability function di(yi). Desirability function is linear if r = 1 and convex when r > 1. If the value of r lies between 0 and 1 the shape of di(yi) is concave. The individual desirability index of all the responses are then combined using the geometric mean to get the overall desirability or composite desirability Di:

\[
D_i = \left[ d_i(y_1)^{w_1} \times d_i(y_2)^{w_2} \times d_i(y_3)^{w_3} \times \ldots \times d_i(y_m)^{w_m} \right]^{1/m} \quad (12)
\]

Where, di (i = 1, 2, 3… m) is the individual desirability of the response, m is the weight of di and W is the sum of the individual weights.

4. RESULTS AND DISCUSSION

4.1 Experimental details

The experiments have been conducted on a high speed three axis CNC vertical machining center (Model: MICRON VCP 710, Germany). All the test runs have been performed in dry environment. A 2-fluted solid carbide ball end milling cutter (Sandvik coromant, CoroMill Plura, R216.42-10030-A110G 1620) with PVD monolayer coating of TiAlN has been used. The diameter, rake angle and helix angle of the cutter is 10 mm, 4° and 30° respectively. The experiments have been performed on aluminium alloy AI2014-T6 work piece block of size 160×100×100 mm³. The cutting forces in FX, FY and FZ directions have been measured with Kistler 3-component piezoelectric dynamometer (Model: 9257B) along with multichannel charge amplifier (Type: 5011B) and a data acquisition system. The dynamometer was calibrated with static and dynamic loading before starting the experiments. Cutting force data have been collected at the sampling rate of 1200 Hz and the post processing of the cutting force data have been performed using Dynoware software in a computer with windows XP as operating system.

Dynamic cutting forces in FX, FY and FZ directions are obtained by slot milling tests performed according the experimental plan as shown in Table 2. Noises in the cutting force signals have been eliminated during the post processing and maximum cutting forces in FX, FY and FZ directions have been obtained. Thus, the cutting force signals in these three directions have been again post processed and subsequently, dynamic force components have been determined as mentioned earlier. A full quadratic model is considered and effect of each cutting parameters have been studied for each of the responses, i.e., FX, FY and FZ. MINITAB 16 has been used for the above purpose. Thus, the regression equations have been determined. The significance tests for the input parameters as well as for the models have been carried out and the model adequacy has been checked.

4.2. Analysis of variance (ANOVA) and mathematical model development

Significance tests for individual terms have been performed through ANOVA. Table 3 shows the ANOVA and fit summary for dynamic tangential cutting force component. The significant terms are characterized by the p-value less than 0.05 (i.e. α = 0.05 or 95% level of confidence). It may be observed that the linear terms of the model are significant, whereas some of the squared terms and interaction terms like, fj*fj, aj*aj, fj*V, aj*V and aj*V are insignificant. The insignificant terms have been eliminated through backward elimination process one by one to reduce the model and in every step adequacy of the model has been checked. Thus, ANOVA and fit summary of the final model for FX is shown in the Table 4. Table 4 also reveals that the second order regression model is highly significant and also the lack-of-fit is insignificant. An insignificant lack-of-fit is desirable. R² is one of the most important statistic which is a measure of the amount of reduction in the variability of response obtained by using the regressor variables in the model. As R² approaches to unity, the response model fits better with the actual data. However, a large value of R² does not necessarily imply that the regression model is good. Adding a variable to the model will always increase in R² as it always increases when terms are added to the model.
Table 2. Experimental design layout and results for cutting force component

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</table>

Table 3. ANOVA and fit summary of tangential dynamic cutting force ($F_X$)

Therefore, adjusted $R^2$ ($R^2$-adj) is preferred. It often decreases if unnecessary terms are added [12]. When the difference between $R^2$ and $R^2$-adj is large, there would be a good chance that insignificant terms have been included in the model.

From Table 4, it is seen that the difference between $R^2$ and $R^2$-adj is very small (0.0062), which is desirable. The obtained $R^2$-adj value (0.9792) is very close to unity, which shows a very good correlation between experimental and predicted results. The adequacy of the model has been checked using the F-test also. The F-value for the lack of fit of developed model should not exceed the standard tabulated F-value. A very high F-value (157.58) and a very small p-value (negligibly small) for the model shows that the model is adequate. The F-value of lack-of fit of the developed quadratic model is 3.05, which is less than the standard tabulated F-value (4.06) at 95% confidence level. Thus, the model is sufficiently good to be acceptable.
Table 5. ANOVA and fit summary of radial dynamic cutting force (F_r) for reduced model

<table>
<thead>
<tr>
<th>Sources</th>
<th>DoF</th>
<th>Seq SS</th>
<th>F</th>
<th>p</th>
<th>PC (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>10</td>
<td>38567.7</td>
<td>157.58</td>
<td>0.000</td>
<td>98.56</td>
</tr>
<tr>
<td>f_r</td>
<td>1</td>
<td>9160.0</td>
<td>336.83</td>
<td>0.000</td>
<td>23.40</td>
</tr>
<tr>
<td>a_p</td>
<td>1</td>
<td>192865.7</td>
<td>709.21</td>
<td>0.000</td>
<td>49.27</td>
</tr>
<tr>
<td>a_e</td>
<td>1</td>
<td>4760.7</td>
<td>171.75</td>
<td>0.000</td>
<td>12.16</td>
</tr>
<tr>
<td>V</td>
<td>1</td>
<td>28866.6</td>
<td>106.15</td>
<td>0.000</td>
<td>7.3</td>
</tr>
<tr>
<td>a_r*a_e</td>
<td>1</td>
<td>1021.1</td>
<td>34.54</td>
<td>0.000</td>
<td>2.60</td>
</tr>
<tr>
<td>f_r*V</td>
<td>1</td>
<td>182.5</td>
<td>6.71</td>
<td>0.017</td>
<td>0.46</td>
</tr>
<tr>
<td>f_r*a_p</td>
<td>1</td>
<td>436.3</td>
<td>16.04</td>
<td>0.001</td>
<td>1.12</td>
</tr>
<tr>
<td>f_r*a_e</td>
<td>1</td>
<td>576.4</td>
<td>21.19</td>
<td>0.000</td>
<td>1.47</td>
</tr>
<tr>
<td>a_p*a_e</td>
<td>1</td>
<td>347.5</td>
<td>12.78</td>
<td>0.002</td>
<td>0.88</td>
</tr>
<tr>
<td>Residual Error</td>
<td>6</td>
<td>571.1</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Lack-of-Fit</td>
<td>15</td>
<td>504.9</td>
<td>3.05</td>
<td>0.088</td>
<td>Insignificant</td>
</tr>
<tr>
<td>Pure error</td>
<td>6</td>
<td>66.1</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>30</td>
<td>39138.8</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4. ANOVA and fit summary of tangential dynamic cutting force (F_t) for reduced model

Similar procedure has been employed for F_Y, F_Z dynamic cutting force components. ANOVA and fit summary for the reduced quadratic models are shown in the Table 5 and Table 6 respectively. Thus, the second order mathematical models for F_X, F_Y, F_Z in coded forms are determined as:

\[ F_X = 96.50 + 19.53 f_r + 28.34 a_p + 13.95 a_e - 10.96 V - 5.67 a_r^2 + 2.50 f_r^2 a_p + 6.00 f_r a_e + 4.66 a_r a_e \]  \hspace{1cm} (13)

\[ F_Y = 23.62 f_r + 3.06 a_p + 11.00 a_e - 4.84 V + 0.86 a_r^2 + 0.97 f_r^2 \]  \hspace{1cm} (14)

\[ F_Z = 48.00 f_r + 12.31 a_p + 7.01 a_e - 6.93 V - 1.92 a_r^2 + 1.49 f_r^2 a_p + 1.61 f_r a_e + 3.69 a_r a_e \]  \hspace{1cm} (15)

Table 6. ANOVA and fit summary of axial dynamic cutting force (F_a) for reduced model

4.3. Analysis of the influences of cutting parameters on the responses

Percentage contribution (PC) of each term for F_X is shown in the Table 4. The main effect of a_p is most significant with a PC of 49.27% in linear part followed by f_r 23.40%, a_e 12% and V 7.3% respectively. The second order effect of radial depth of cut is more significant than cutting speed. The two level interaction of feed per tooth and radial depth of cut (f_r*a_p) is most significant. From the second order model, it is evident that dynamic cutting force in F_X increases with increase in a_p and f_r. It decreases with increase in V. From Table 5, it is revealed that F_Y is mostly affected by a_p with a PC of 64.21% followed by V and f_r. In two-level interaction a_e is also the most important factor. F_Z component of the dynamic force shows similar behavior like F_X. Axial depth of cut is most dominant factor among all the cutting parameters with the total effect of 45.10 % in linear terms followed by feed per tooth and cutting speed. It is interesting to note that the PC of cutting speed and radial depth of cut is almost same as shown in Table 6. Values of all the three components of the dynamic cutting force decrease with increase in cutting speed. F_X and F_Z are mostly affected by cutting speed whereas F_Y is affected very little by it. All the dynamic cutting forces (F_X, F_Y and F_Z) increase with increase in axial depth of cut. 3-D surface graphs for tangential, axial and radial components of the dynamic force are shown in Figs. 2-4 respectively. All the three components have curvilinear profile in accordance with the fitted quadratic model. From the Figs. 2-4, it is clear that the cutting force components increase with increase in axial depth of cut and feed per tooth.
Figs. 2 (b) and (d), we can observe that $F_X$ decreases with the increase in $V$. In case of other parameters like $f_z$, $a_p$, and $a_e$, similar trend has been observed for $F_Z$ as shown in the Figs. 3 (a-d).

![Fig. 2. Response surface plots for tangential cutting force component ($F_X$)](image1)

![Fig. 3. Response surface plots for axial cutting force component ($F_Z$)](image2)
Contour and surface plots of $F_Y$ is shown in Figs. 4 (a) and (b). From Fig. 4 (a), it is evident that the $a_e$ is the most dominating cutting parameters. $F_Y$ increases with increase in the value of $a_e$.

Predicted values of dynamic force components ($F_X$, $F_Y$ and $F_Z$) obtained from the quadratic model (at 95% confidence level) are very close to the experimental values as shown in Fig. 5. The results show that there is a variations of 6.12%, -7.18% and 8.31% in tangential, radial and axial dynamic force components, respectively in the cutting speed range of 75-150 m/min and axial depth of cut range of 0.2-1.4 mm. The variation in predicted result is mostly observed at higher cutting speed of 150 m/min and axial and radial depth of cut ranging from 1-1.4 mm and 0.5-0.7 mm respectively. At lower depth of cut the variation is very small accounting 3.58%, -2.16% and 5.23% in tangential radial and axial components shown in Fig 5.

4.4 Optimization results using CD

In the present study, the goal is to minimize the dynamic cutting forces components. Optimization result for dynamic cutting force components using composite desirability function (CD) is shown in Fig. 6. In Fig. 6, column represents cutting parameters while a row corresponds to responses. Variation of responses are indicated by each cell of the plot with change of a parameter keeping other parameters fixed. The current parameter settings are indicated at middle row of the top of the column and can change with the parameters settings interactively. The responses (dynamic cutting force components, $F_X$, $F_Y$ and $F_Z$) goal for the response minimization, predicted values of responses at the current parameter settings with individual desirability score are indicated at each row in left column. The optimum results for $F_X$, $F_Y$ and $F_Z$ dynamic cutting force components using RSM optimization is shown in the Table 7. The optimum cutting parameters in coded unit are found to be feed per tooth of at level -1.01 (0.07 mm/tooth), axial depth of cut -0.45 (0.82 mm), radial depth of cut -0.67 (0.53 mm) and cutting speed at level of 1.56 (164 m/min). Corresponding optimized dynamic force components are $F_X = 48.62$ N, $F_Y = 11.25$ N and $F_Z = 25.18$ N respectively. Overall composite desirability is 0.93 shown in Fig. 6.

4.5 Optimization results using TLBO

The detailed procedures for implementation of TLBO are stated as follows:
(1) Defining the optimization problem as:

\[
\text{Minimize } R \left( f_z, a_p, a_e, V \right) \\
\text{Subjected to:}
\]

\[
0.02 \text{ mm/tooth} \leq f_z \leq 0.22 \text{ mm/tooth}, \quad 0.2 \text{ mm} \leq a_p \leq 1.8 \text{ mm}
\]

\[
0.1 \text{ mm} \leq a_e \leq 0.9 \text{ mm}, \quad 75 \text{ m/min} \leq V \leq 175 \text{ m/min}
\]

The developed mathematical model for dynamic cutting forces have been used as fitness function of TLBO for minimization. The constraints are $f_z$, $V$, $a_p$ and $a_e$.

(2) Initialize the population.

In the second step, a random population is generated according to population size and number of cutting parameters. Learners represent population size and subjects represent the cutting process parameters in TLBO. The cutting process parameters are used to generate a random initial population.

(3) Teacher phase solution

(4) Learner phase solution

(5) Termination criterion.

The program will be terminated when the maximum generation number is achieved, otherwise it will again start from step 3 and will continue till the maximum generation number is achieved. In the present study, Deb’s heuristic constraint handling method is adopted to handle the constraints in the problem. Initially various trails were carried out by running TLBO algorithm for different population size, numbers of generations and
teaching factor to get the consistent results. The optimum results have been obtained at 20 population size, 500 number of generations and in 100 iterations. Teaching factor was selected 1 after numerous trails. The obtained result is shown in the Table 7. From Table 7, it may be seen that the optimal cutting forces obtained by TLBO are better than those obtained from CD.

<table>
<thead>
<tr>
<th>OPM</th>
<th>Optimum combinations</th>
<th>PR (N)</th>
<th>Dc</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>R</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CD</td>
<td>Fz -1.01 -0.45 -0.67 1.56 48.62 0.95</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fx -1.23 -0.52 -0.43 1.76 46.70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TLBO</td>
<td>Fz 1.23 -0.52 -0.43 1.76 24.84</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fx 1.23 -0.52 -0.43 1.76 24.84</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

OM-Optimization methods; R-Responses; PR-predicted responses; Dc-composite desirability
Table 7. Optimal value of process parameters (coded value)

**4.6 Confirmation experiments**

Confirmation experiments are carried out to with the set of cutting parameters obtained using CD and TLBO demonstrate the effectiveness of the multi-objective optimization approaches. Two test conditions are also selected from the previous experimental plan (from Table 2), one of the conformation test is performed by selecting random parametric value in the range of cutting parameters. That combination of cutting parameters are further used in the response equations of Fx, Fy and Fz to verify the developed mathematical models. Conformation experiment settings are shown in Tables 8. In Table, the number in parenthesis represents the coded level of the conformation experiments. Predicted and experimental values dynamic cutting force components are shown in Table 9. It is found that the experimental values of the optimized cutting parameters are very close to predicted results.

**Table 8. Parameter settings for confirmation experiments**

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Exp. No.</th>
<th>f_z (mm/tooth)</th>
<th>a_p (mm)</th>
<th>a_e (mm)</th>
<th>V (m/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>OC*</td>
<td>0.07</td>
<td>0.82</td>
<td>0.53</td>
<td>164</td>
</tr>
<tr>
<td></td>
<td>(CD)</td>
<td>(-1.04)</td>
<td>(-.45)</td>
<td>(-0.67)</td>
<td>(1.56)</td>
</tr>
<tr>
<td>2</td>
<td>OC*</td>
<td>0.06</td>
<td>0.79</td>
<td>0.42</td>
<td>169</td>
</tr>
<tr>
<td></td>
<td>(TLBO)</td>
<td>(-1.23)</td>
<td>(-0.52)</td>
<td>(-0.43)</td>
<td>(1.76)</td>
</tr>
<tr>
<td>3</td>
<td>IC*</td>
<td>0.07</td>
<td>1.4</td>
<td>0.3</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-1)</td>
<td>(1)</td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Exp. No.</td>
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<td>1.4</td>
<td>0.7</td>
<td>150</td>
</tr>
<tr>
<td></td>
<td>16</td>
<td>(1)</td>
<td>(1)</td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Exp. No.</td>
<td>0.12</td>
<td>1</td>
<td>0.9</td>
<td>125</td>
</tr>
<tr>
<td></td>
<td>22</td>
<td>(0)</td>
<td>(0)</td>
<td>(2)</td>
<td>(1)</td>
</tr>
</tbody>
</table>

*OC-optimal combination; IC-intermediate combination

Table 9. Results of confirmation experiments

**5. CONCLUSIONS**

In the present work, an attempt has been made to determine the dynamic force components of the cutting forces obtained from the slot milling test in high speed ball end milling process of Al2014-T6. Dynamic cutting forces in Fx, Fy and Fz directions have been determined using Eqs. (1 and 2) using 20000 sample points. Machining experiments have been planned using central composite design. Analysis of variance (ANOVA) has been performed to investigate the effect of cutting process parameters on the dynamic forces and mathematical models have been developed. Developed mathematical models are further used as fitness function to get optimal combinations of cutting parameters using teaching learning based optimization and composite desirability function. Following conclusions have been drawn from the analysis of the results in the present study:

1) Tangential and axial cutting force components are influenced mainly by axial depth of cut with percentage contributions of 49.27% and 45.10%, respectively. Increase in axial depth of cut increases the dynamic cutting force components. Radial cutting force component is mostly affected by radial depth of cut with a percentage contribution of 64.21%.

2) Cutting speed has significant effect on all the three dynamic cutting force components and increase in cutting speed decreases the magnitude of force component values.

3) Composite desirability function and a comparatively new optimization method TLBO have been applied for optimizing the cutting parameters for all the three...
dynamic cutting force components. It is found that TLBO provides better results than composite desirability function approach.  
4) The optimization using TLBO methodology took minimum effort and less computational time. The optimal parametric setting obtained by TLBO has been verified through experiments. Thus, the method is an efficient method and may further be employed for optimization of cutting forces and material removal rate in high speed ball end milling process.  
5) Confirmation tests have been conducted with optimal cutting parameters to verify the optimization results and effectiveness of the multi-objective optimization approach. It is found that the experimental values at optimized cutting parameters are very close to the predicted results.

6. REFERENCES


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